Efficient polarization entanglement purification based on parametric down-conversion sources with cross-Kerr nonlinearity*

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We present a way for entanglement purification based on two parametric down-conversion (PDC) sources with cross-Kerr nonlinearities. It is comprised of two processes. The first one is a primary entanglement purification protocol for PDC sources with nondestructive quantum nondemolition (QND) detectors by transferring the spatial entanglement of photon pairs to their polarization. In this time, the QND detectors act as the role of controlled-not (CNot) gates. Also they can distinguish the photon number of the spatial modes, which provides a good way for the next process to purify the entanglement of the photon pairs kept more. In the second process for entanglement purification, new QND detectors are designed to act as the role of CNot gates. This protocol has the advantage of high yield and it requires neither CNot gates based on linear optical elements nor sophisticated single-photon detectors, which makes it more convenient in practical applications.

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I. INTRODUCTION

Quantum entanglement plays an important role in quantum information processing and transmission, such as quantum computation [1], quantum teleportation [2], quantum dense coding [3], quantum state sharing [4] and certain types of quantum cryptography [5, 6, 7, 8, 9, 10]. In order to complete these tasks efficiently, people need to share some maximally entangled states. In a practical transmission, the interaction between a quantum system and the innocent noise of quantum channel (such as optical fibers or a free space) will inevitably occur, which will degrade the entanglement of the quantum system or even make it in a mixed state. The impurity of the quantum system will make the outcome of the quantum computation anamorphic, the fidelity of quantum teleportation degraded, quantum dense coding failed and the key in quantum cryptography insecure. If the destructive effect of the noise is not very much, one can exploit entanglement concentration or entanglement purification to improve the entanglement of the quantum system first, and then achieve the goals of the applications above with maximally entangled state.

Entanglement concentration [11, 12, 13, 14] is used to increase the entanglement of some pure entangled pairs at the risk of that of some others. For the more general case of the quantum system transmitted through a noisy channel, it is in a mixed state and the process for reconstructing it in a maximally entangled state with an ensemble is termed as entanglement purification or distillation [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Generally, the implementation of entanglement purification schemes requires two or more controlled-not (CNot) gates which is not experimentally feasible with linear optical elements at present. In 1996, Bennett *et al.* [17] proposed an original entanglement purification scheme for purifying a Werner state [26] with two CNOT gates and single-photon measurements. Subsequently, Deutsch *et al.* [16] optimized this scheme for quantum privacy amplification with two CNOT operations and two special unitary transforms.

In 2001, Pan *et al.* [18] proposed an entanglement purification protocol with linear optical elements such as polarizing beam splitters (PBSs) and quarter wave plates (QWPs). In their protocol [18], the two PBSs are used to complete the task of parity-check measurements of polarized photons with their spatial modes. We call it PBS protocol below. This protocol succeeds, provided that two ideal entangled sources are used. That is, both emit one and only one entangled photon pair synchronously at each time slot. As pointed out by Simon and Pan [27] in 2002, the currently available source of entangled photons, parametric down-conversion (PDC), is not an ideal entangled source. The feature of PDC seems to fail for the PBS protocol [18]. They then proposed a new entanglement purification protocol by exploiting spatial entanglement to purify polarization entanglement, which solves the problem above perfectly [27], and called it Simon-Pan protocol. However, in order to improve the fidelity of the entangled pairs kept more with the PBS protocol [18], the two parties should exploit quantum nondemolition (QND) measurement to determine whether there are photons after the PBS or not, which can not be ac-

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complished only with PBS. Moreover, photon number detectors should be used to distinguish the two-photon cases from the cases with four photons in the same modes such as two photons in the upper modes of both Alice's and Bob's location. This task can not be accomplished simply with linear optical elements.

Cross-Kerr nonlinearity provides a good tool to construct nondestructive quantum nondemolition detectors "which have the potential available of being able to condition the evolution of our system but without necessarily destroying the single photons" [28, 29]. QND with a cross-Kerr medium and a coherent state can be used for checking the parity of the polarizations of two photons [28], operating as a controlled-not (CNOT) gate [28], and analyzing the Bell states [30]. The Hamiltonian of a cross-Kerr nonlinear medium can be described by the form as follows:

$$H_{QND} = \hbar\chi \hat{n}_a \hat{n}_c \qquad (1)$$

where $\hat{n}_a$ ($\hat{n}_c$) denotes the number operator for mode a (c) and $\hbar\chi$ is the coupling strength of the nonlinearity, which is decided by the property of material. For example, for a signal photon state $|\varphi\rangle = a|0\rangle + b|1\rangle$ and a coherent state $|\alpha\rangle$, the cross-Kerr interaction causes the combined system composed of a single photon and a coherent state to evolve as [28]

$$\begin{aligned} U_{ck}|\varphi\rangle|\alpha\rangle &= e^{iH_{QND}t/\hbar}(a|0\rangle + b|1\rangle)|\alpha\rangle \\ &= a|0\rangle|\alpha\rangle + b|1\rangle|\alpha e^{i\theta}\rangle. \end{aligned} \qquad (2)$$

We note that $|0\rangle$ and $|1\rangle$ are not the polarization of the photons, but the number of the photons. $|n\rangle$ is also called the Fock state which means the state contains n photons. Now one can see that the signal photon state is unaffected by the interaction, but the coherent state makes a phase shift of θ. Here $\theta = \chi t$ and t is the interaction time. The phase shift is directly proportional to the number of photons. This is the main principle of the cross-Kerr nonlinearity [28]. In 2005, Song *et al.* [31] presented a protocol for entanglement purification using cross-Kerr nonlinearity to complete parity check. It works for the original entanglement purification model proposed by Bennett et al. [17]. The biggest advantage of their protocol is that its successful probability can be nearly enhanced to a quantity twice as large as that of PBS protocol [18]. The drawback of this protocol is the same as that in PBS protocol [18]. That is, it requires that the two parties of quantum communication should be in possession of two ideal single-pair entangled sources. Considering the currently available source of entangled photons, this protocol becomes useless. Also, it cannot get perfectly entangled photon pairs purified when the two parties get a nonzero phase shift with X homodyne measurements on their coherent states, which takes place with a probability of the same order of magnitude for the case where Alice and Bob both get the phase shift 0. Moreover, it can not complete the iteration of the purification steps efficiently for improving the fidelity of the entangled photons more.

In this paper, we present a way for entanglement purification based on two PDC sources with cross-Kerr nonlinearities. The task of entanglement purification can be completed with two steps. First, we provide a primary entanglement purification protocol for PDC sources with QND detectors by transferring the spatial entanglement of photon pairs to their polarization. In this protocol, the QND detectors act as not only the role of CNot gates but also that of photon number detectors, which provides a good way for the next process to purify the entanglement of the photon pairs more as they make the photon pairs equivalent to those coming from two ideal sources. In the second process for entanglement purification, new QND detectors are designed to act as the role of CNOT gates. This protocol has the advantage of high yield and it requires neither CNOT gates based on linear optics nor sophisticated single-photon detectors, which makes it more convenient in practical applications.

II. ENTANGLEMENT PURIFICATION BASED ON PDC SOURCES

A. The principle of primary entanglement purification based on bit-flipping errors with QND

The principle of our entanglement purification protocol is shown in Fig.1. The PDC sources can produce polarization and spatial entanglement naturally [27]. A pump pulse of ultraviolet light passes through a beta barium borate (BBO) crystal and produces correlated pairs of photons into the modes a_1 and b_1. Then it is reflected and traverses the crystal a second time, and produces correlated pairs of photons into the modes a_2 and b_2. The Hamiltonian can be approximately described as

$$\begin{aligned} H_{PDC} &= \gamma[(a^+_{1H}b^+_{1H} + a^+_{1V}b^+_{1V}) \\ &+ re^{i\phi}(a^+_{2H}b^+_{2H} + a^+_{2V}b^+_{2V})] + H.c, \end{aligned} \qquad (3)$$

where H and V in subscripts present horizontal and vertical polarization, r denotes the relative probability of emission of photons into the lower modes compared to the upper modes, and ϕ is the phase between these two possibilities [27]. The same as the Simon-Pan protocol [27], in a simple case we assume $r = 1$ and $\phi = 0$. So the single-pair state can be described by $(a^+_{1H}b^+_{1H} + a^+_{1V}b^+_{1V} + a^+_{2H}b^+_{2H} + a^+_{2V}b^+_{2V})|0\rangle$. It also can be written as $(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)(|H_a\rangle|H_b\rangle + |V_a\rangle|V_b\rangle)$. The four-photon state produced by this PDC source also can be written as $(a^+_{1H}b^+_{1H} + a^+_{1V}b^+_{1V} + a^+_{2H}b^+_{2H} + a^+_{2V}b^+_{2V})^2|0\rangle$ and discussed in the same way.

After receiving the signals, the user Alice (Bob) lets them pass through QND_1 detectors whose principle is shown in Fig.2. For a two-photon state without suffering from decoherence (including bit-flipping and phase-flipping) $(a^+_{1H}b^+_{1H} + a^+_{1V}b^+_{1V} + a^+_{2H}b^+_{2H} + a^+_{2V}b^+_{2V})|0\rangle$, the two parties Alice and Bob will get the same phase shifts on their coherent states as QND_1 detectors evolve the

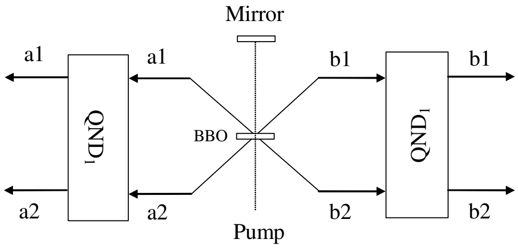

FIG. 1: The new entanglement purification protocol that uses new QND (QND$_1$) detectors and two parametric down-conversion sources. The difference between this protocol and Simon-Pan protocol [27] is that we replace the two PBSs in the latter with two QND$_1$ detectors. The PDC sources, which produce two photons each into modes a1 and b1 and no photons into modes a2 and b2 or vice versa, can be purified by both Alice and Bob selecting the same phase shifts.

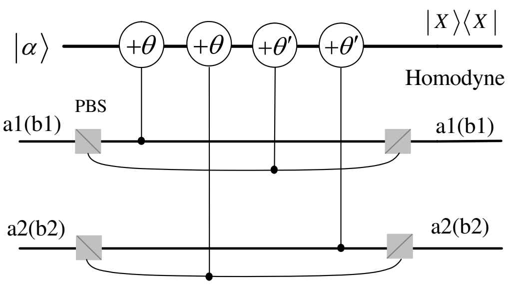

FIG. 2: Schematic diagram showing the principle of our new nondestructive quantum nondemolition (QND$_1$) detectors. Several cross-Kerr nonlinearities and a coherent laser probe beam $|\alpha\rangle$ are used in our protocol. This QND can transform spatial entanglement into polarization entanglement. It can also distinguish superpositions and mixtures of the states $|HH\rangle$ and $|VV\rangle$ from $|VH\rangle$ and $|HV\rangle$. It acts as not only the role of CNot gates but also of photon number detectors here.

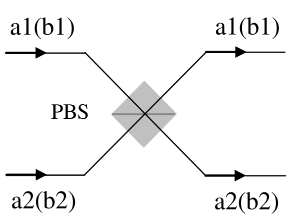

FIG. 3: Schematic diagram showing the principle of a coupler.

combined system to

$$\begin{aligned}\rightarrow\ & (a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})|0\rangle|\alpha e^{i\theta}\rangle_a|\alpha e^{i\theta}\rangle_b \\ & +(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})|0\rangle|\alpha e^{i\theta'}\rangle_a|\alpha e^{i\theta'}\rangle_b, \end{aligned}\tag{4}$$

where $\theta\neq\theta'\oplus 2\pi$. If Alice and Bob get the same results with an X homodyne measurement (θ or θ'), they retain the pair and perform no local unitary operations on their photons but link the photons with couplers shown in Fig.3. If both Alice and Bob get the phase shift θ (θ'), their photon pair in the state $(a^+_H b^+_H + a^+_V b^+_V)|0\rangle$ will appear at the lower output modes a_2b_2 (upper modes a_1b_1) of the couplers. If a bit-flipping error takes place, i.e., the state of the pair becoming $(|a_1\rangle|b_1\rangle+|a_2\rangle|b_2\rangle)(|V_a\rangle|H_b\rangle+|H_a\rangle|V_b\rangle)$, Alice and Bob will get two different results with their homodyne measurements on their coherent states $|\alpha\rangle$ as QND$_1$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & (a^+_{1V}b^+_{1H}+a^+_{2H}b^+_{2V})|0\rangle|\alpha e^{i\theta'}\rangle_a|\alpha e^{i\theta}\rangle_b \\ & +(a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H})|0\rangle|\alpha e^{i\theta}\rangle_a|\alpha e^{i\theta'}\rangle_b. \end{aligned}\tag{5}$$

One will get the result θ and the other θ'. Therefore, by performing a bit-flipping operation $\sigma_x=|H\rangle\langle V|+|V\rangle\langle H|$, Alice and Bob can get rid of all bit-flip errors and obtain their uncorrupted pairs $(a^+_H b^+_H + a^+_V b^+_V)|0\rangle$ by coupling the two spatial modes with their couplers.

Certainly, a phase-flipping error can not be directly purified in this way. However, as pointed out by others [15, 16, 17, 18], a phase-flipping error can be transformed into a bit-flipping error using a bilateral local operation. If a bit-flipping error purification has been successfully solved, phase-flipping errors also can be solved perfectly. In this way the two parties can purify a general mixed state. We only discuss the case with bit-flipping errors below.

For the four-photon state $(a^+_{1H}b^+_{1H}+a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H}+a^+_{2V}b^+_{2V})^2|0\rangle$ which has the same order of magnitude of probability as the two-photon state $(a^+_{1H}b^+_{1H}+a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H}+a^+_{2V}b^+_{2V})|0\rangle$, if it does not suffer from decoherence, the QND$_1$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & (a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})^2|0\rangle|\alpha e^{i2\theta}\rangle_a|\alpha e^{i2\theta}\rangle_b \\ & +(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})^2|0\rangle|\alpha e^{i2\theta'}\rangle_a|\alpha e^{i2\theta'}\rangle_b \\ & +2(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})(a^+_{2H}b^+_{2H} \\ & +a^+_{1V}b^+_{1V})|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b. \end{aligned}\tag{6}$$

Similar to the case with the two-photon state, Alice and Bob will get the same phase shifts with their homodyne measurements on their coherent states. That is, they both get 2θ, $2\theta'$, or $\theta+\theta'$ which corresponds to the four-photon state $(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})^2|0\rangle$, $(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})^2|0\rangle$, or $(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})(a^+_{2H}b^+_{2H}+a^+_{1V}b^+_{1V})|0\rangle$, respectively. The state $(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})(a^+_{2H}b^+_{2H}+a^+_{1V}b^+_{1V})|0\rangle$ represents the case that one pair appears at the upper modes and the other at the lower modes after the couplers, and both in the desired state $(a^+_H b^+_H + a^+_V b^+_V)|0\rangle$. The state $(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})^2|0\rangle$ ($(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})^2|0\rangle$) denotes that the two pairs both appear at the lower (upper) modes after the couplers. That is, the QND$_1$ detectors can pick up the state wanted from others with the spatial entanglement resource.

If a bit-flipping error takes place on one of the two photon pairs in the four-photon state, i.e., the state of the two photon pairs becoming $(a^+_{1H}b^+_{1H}+a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H}+$

$a^+_{2V}b^+_{2V})(a^+_{1V}b^+_{1H}+a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H}+a^+_{2H}b^+_{2V})|0\rangle$, the QND$_1$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\; & (a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})(a^+_{1V}b^+_{1H}+a^+_{2H}b^+_{2V})|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i2\theta}\rangle_b\\ & +(a^+_{1H}b^+_{1H}+a^+_{2V}b^+_{2V})(a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H})|0\rangle|\alpha e^{i2\theta}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b\\ & +(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})(a^+_{1V}b^+_{1H}+a^+_{2H}b^+_{2V})|0\rangle|\alpha e^{i2\theta'}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b\\ & +(a^+_{1V}b^+_{1V}+a^+_{2H}b^+_{2H})(a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H})|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i2\theta'}\rangle_b. \end{aligned}\tag{7}$$

That is, Alice and Bob can not get the same phase shifts with their homodyne measurements on their coherent states. After the couplers, one of the two parties gets two photons coming from two modes but the other gets two photons from only one mode, which makes Alice and Bob have no ability to get the uncorrupted state $(a^+_H b^+_H + a^+_V b^+_V)|0\rangle$ perfectly. Alice and Bob discard all these instances, the same as the Simon-Pan protocol [27].

If bit-flipping errors take place on both the two photon pairs in the four-photon state, i.e., the state of the two pairs becoming $(a^+_{1V}b^+_{1H}+a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H}+a^+_{2H}b^+_{2V})^2|0\rangle$, QND$_1$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\; & (a^+_{1H}b^+_{1V}+a^+_{2V}b^+_{2H})^2|0\rangle|\alpha e^{i2\theta}\rangle_a|\alpha e^{i2\theta'}\rangle_b\\ & +(a^+_{1V}b^+_{1H}+a^+_{2H}b^+_{2V})^2|0\rangle|\alpha e^{i2\theta'}\rangle_a|\alpha e^{i2\theta}\rangle_b\\ & +2(a^+_{1V}b^+_{1H}+a^+_{2H}b^+_{2V})(a^+_{1H}b^+_{1V}\\ & +a^+_{2V}b^+_{2H})|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b. \end{aligned}\tag{8}$$

Alice and Bob should discard the instances for which one gets the phase shift 2θ and the other gets $2\theta'$ as the two photon pairs appear at the same mode simultaneously. When both get the phase shift $\theta+\theta'$, they will keep these unwanted photon pairs as they can not distinguish the corrupted photon pairs from the uncorrupted ones in this way.

In a practical application, the spatial entanglement in this two-photon state is completely transformed into the polarization entanglement in the process for eliminating the bit-flipping errors. They can not be used again for correcting the phase-flipping errors directly. It requires the two parties to exploit another purification protocol to solve this problem. It is valuable to point out that the QND$_1$ detectors act as not only a nondestructive measurement tool but also a tool for distinguishing the number of photons. This tool is very useful for the next purification to improve the fidelity of the pairs more.

B. The fidelity of the photon pairs

Now let us pay our attention to the fidelity of the photon pairs. Suppose the probabilities of one photon pair and two photon pairs produced by PDC sources are p_1 and p_2, respectively. Suppose the probability of bit-flipping arisen from the quantum channel such as fibers is $1-F_0$, which means the original fidelity of the photon pairs controlled by the two users Alice and Bob is F_0.

In our primary entanglement purification protocol, the two-photon states are all be kept, which takes place with the probability p_1, and their fidelity is 1 after Alice performs a bit-flipping operation or not. For the four-photon states, Alice and Bob only keep the cases where each mode has one and only one photon, which takes place with the probability of $\frac{1}{2}p_2[F_0^2+(1-F_0)^2]$. After the primary entanglement purification, the fidelity of the photon pairs kept becomes

$$F_1=\frac{p_1+\frac{1}{2}p_2F_0^2}{p_1+\frac{1}{2}p_2[F_0^2+(1-F_0)^2]}. \tag{9}$$

III. ENTANGLEMENT PURIFICATION BASED ON IDEAL SOURCES

After the primary entanglement purification based on PDC sources in Sec. II, the photon pairs kept are equivalent to those coming from two ideal sources as the QND$_1$ can distinguish the two-photon states from the four-photon states. Moreover, it shows there are useful photon pairs or not clearly for the two users. In this time, Alice and Bob can exploit the entanglement purification protocols with CNOT gates such as those in Refs. [16, 17] or the PBS protocol proposed by Pan et al. [18] to improve the fidelity of the photon pairs more. At present, a CNOT gate with single photons is far beyond what is experimentally feasible. The PBS protocol requires sophisticated single-photon detectors and its yield of photon pairs purified is only half of that with CNOT gates. The protocol in Ref. [31] with QND detectors designed by Nemoto and Munro [28] can also be used to purify less entangled pairs with X quadrature measurements [32] in a nearly deterministic way as the two users Alice and Bob should ensure that the states $|\alpha e^{\pm i\theta}\rangle$ can not be distinguished.

In this section, we will present a different entanglement purification protocol for ideal sources in a completely deterministic way without CNOT gates and sophisticated single-photon detractors. It has the same yield of photon pairs purified as those [16, 17] with CNOT gates, double that of the PBS protocol [18].

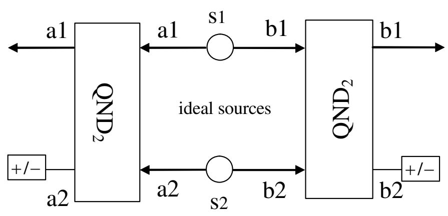

FIG. 4: The new entanglement purification protocol that uses new QND (QND_2) detectors and two ideal sources (S1 and S2). It can be used as the second purification process for improving the fidelity of photon pairs from PDC sources more.

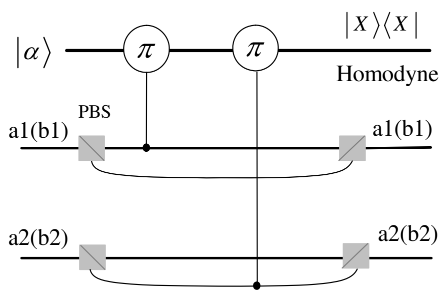

FIG. 5: Schematic diagram showing the principle of our new nondestructive quantum nondemolition detectors (QND_2). Each QND_2 is composed of two same cross-Kerr nonlinearities with the phase shift $\theta = \chi t = \pi$, four PBSs and a coherent laser probe beam $|\alpha\rangle$.

Suppose the photon pairs after the primary entanglement purification are in the mixed state ρ_{ab}, the same as that in Ref.[18], described as follows:

$$\rho_{ab} = F|\Phi^+\rangle_{ab}\langle\Phi^+| + (1-F)|\Psi^+\rangle_{ab}\langle\Psi^+|, \qquad (10)$$

where $|\Phi^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b + |V\rangle_a|V\rangle_b)$ and $|\Psi^+_{ab}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)$. F $(> \frac{1}{2})$ is the fidelity of the state, i.e., $F = \langle\Phi^+|\rho_{ab}|\Phi^+\rangle$. The two photons in the state $|\Phi^+\rangle_{ab}$ have the equal polarizations and those in the state $|\Psi^+_{ab}\rangle$ have the opposite polarizations in the rectangle basis $\{|H\rangle, |V\rangle\}$. The two pairs can be seen as the mixture of four states, i.e., $|\Phi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2}$ with a probability of F^2, both $|\Phi^+\rangle_{a1b1}\cdot|\Psi^+\rangle_{a2b2}$ and $|\Psi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2}$ with an equal probability of $F(1-F)$, and $|\Psi^+\rangle_{a1b1}\cdot|\Psi^+\rangle_{a2b2}$ with a probability of $(1-F)^2$.

The principle of our entanglement purification protocol based on two ideal sources is shown in Fig.4. It is composed of two QND detectors (QND_2) and two measurements with the diagonal basis $\{|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)\}$. The principle of QND_2 is shown in Fig.5. The two same cross-Kerr nonlinearities provide the same phase shift $\theta = \chi' t = \pi$.

Let us first consider the state $|\Phi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2}$.

$$\begin{aligned}|\Phi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2} &= \frac{1}{\sqrt{2}}(|H\rangle_{a1}|H\rangle_{b1} + |V\rangle_{a1}|V\rangle_{b1})\\ &\quad\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a2}|H\rangle_{b2} + |V\rangle_{a2}|V\rangle_{b2})\\ &= \frac{1}{2}(|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2} + |H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}\\ &\quad+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2} + |V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}).\end{aligned} \qquad (11)$$

QND_2 detectors evolve the combined system composed of four photons and two coherent states to

$$\begin{aligned}\rightarrow\ &\frac{1}{2}\{(|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}\\ &+|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2})|\alpha e^{i\pi}\rangle_a|\alpha e^{i\pi}\rangle_b\\ &+|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|\alpha e^{i2\pi}\rangle_a|\alpha e^{i2\pi}\rangle_b\\ &+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|\alpha\rangle_a|\alpha\rangle_b\}.\end{aligned} \qquad (12)$$

When both Alice and Bob get the phase shift π with their homodyne measurements on their coherent states, the two photon pairs project to the state $(|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2} + |V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2})$. When they both get the phase shift 0 (2π is just the phase shift 0 for the coherent states), they get the state $(|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2})$ and they can obtain the state $(|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2} + |V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2})$ by performing a bit-flipping operation $\sigma_x = |H\rangle\langle V| + |V\rangle\langle H|$ on their first photons $a1$ and $b1$. With the same way as in Ref.[18] Alice and Bob can make the photon pair in the state $|\Phi^+\rangle_{ab}$. In detail, Alice and Bob first take a measurement with the diagonal basis on their second photons a_2 and b_2. When they both get the results $|+\rangle$ (or $|-\rangle$), the photon pairs a_1b_1 are projected to the state $|\Phi^+\rangle_{ab}$. When one gets the result $|+\rangle$ and the other gets $|-\rangle$, they can obtain the state $|\Phi^+\rangle_{ab}$ by performing the phase-flipping $\sigma_z = |H\rangle\langle H| - |V\rangle\langle V|$ on the photon a_1.

For the cross-combinations $|\Phi^+\rangle_{a1b1}\cdot|\Psi^+\rangle_{a2b2}$ and $|\Psi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2}$, the QND_2 detectors will evolve the combined system to the state in which Alice and Bob can not get the same phase shift with their homodyne measurements on their coherent states. In detail, $|\Phi^+\rangle_{a1b1}\cdot|\Psi^+\rangle_{a2b2}$ will be evolved to

$$\begin{aligned}\rightarrow\ &\frac{1}{2}\{(|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}\\ &+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2})|\alpha\rangle_a|\alpha e^{i\pi}\rangle_b\\ &+|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|\alpha e^{i\pi}\rangle_a|\alpha e^{i2\pi}\rangle_b\\ &+|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|\alpha e^{i\pi}\rangle_a|\alpha\rangle_b\},\end{aligned} \qquad (13)$$

and $|\Psi^+\rangle_{a1b1}\cdot|\Phi^+\rangle_{a2b2}$ will be evolved to

$$\begin{aligned}\rightarrow\ &\frac{1}{2}\{(|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}\\ &+|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2})|\alpha\rangle_a|\alpha e^{i\pi}\rangle_b\\ &+|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|\alpha e^{i\pi}\rangle_a|\alpha e^{i2\pi}\rangle_b\\ &+|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|\alpha e^{i\pi}\rangle_a|\alpha\rangle_b\}.\end{aligned} \qquad (14)$$

When Alice gets the phase shift 0 and Bob gets π, their two photon pairs a_1b_1 and a_2b_2 are in the state $(|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2})$ or $(|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}+|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2})$ with the same probability. In this time, Alice and Bob can not determine in which pair takes place a bit-flipping error. For improving the fidelity of the photon pairs kept, Alice and Bob should discard both these photon pairs, the same as that in the protocol with CNOT gates [16, 17]. When Alice gets the phase shift π and Bob gets 0, they should also discard their two photon pairs.

For the state $|\Psi^+\rangle_{a1b1}\cdot|\Psi^+\rangle_{a2b2}$, QND$_2$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & \frac{1}{2}\{(|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2} \\ & +|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2})|\alpha e^{i\pi}\rangle_a|\alpha e^{i\pi}\rangle_b \\ & +(|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2} \\ & +|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2})|\alpha\rangle_a|\alpha\rangle_b\}. \qquad (15)\end{aligned}$$

When Alice and Bob both get the phase shift π, their two photon pairs are in the state $(|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}+|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2})$. After Alice and Bob perform a measurement with the diagonal basis on their second photons a_2 and b_2, the first photon pair a_1b_1 projects to the state $|\Psi^+\rangle_{ab}$ when they both obtain the outcome $|+\rangle$ (or $|-\rangle$); otherwise Alice and Bob will make the pair a_1b_1 in this state by performing a phase-flipping operation σ_z. When Alice and Bob both get the phase shift 0, their two photon pairs are in the state $(|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}+|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2})$. With the same operations as those in the case where both photon pairs do not contain errors, Alice and Bob will make their first photon pair in the state $|\Psi^+\rangle_{ab}$. In other words, Alice and Bob can not distinguish the two cases that contain no errors in their two photon pairs or that both contain a bit-flipping error. They keep those photon pairs for improving their fidelity in the next round.

By postselection according to the phase shifts of the coherent states, Alice and Bob only keep the first photon pair in the instances where they get the same phase shifts. After this purification process, the new fidelity of the photon pairs kept becomes

$$F'=\frac{F^2}{F^2+(1-F)^2}. \qquad (16)$$

We get the same fidelity as in the PBS protocol [18], but the yield is double that in the PBS protocol as Alice and Bob will keep a photon pair when they get the same phase shift, no matter what it is. In PBS protocol, Alice and Bob only keep the instances that each mode has one and only one photon, which makes its yield half those with CNOT gates [16, 17]. Moreover, Alice and Bob use the homodyne measurements on their coherent states to replace the sophisticated single-photon detectors in PBS protocol [18]. This new entanglement purification protocol can be used to improve the fidelity of photon pairs more by iteration.

IV. DISCUSSION AND SUMMARY

In the primary entanglement purification protocol, Alice and Bob can also use the QND$_3$, whose principle is shown in Fig.6, to purify the photon pairs produced by two PDC sources if they can control accurately the overlap time of the photons coming from the upper mode and the lower mode. In essence, the two parties exploit the cross-Kerr nonlinearities, instead of the sophisticated single-photon detectors in the Simon-Pan protocol [27], to complete the task of distinguishing the photon numbers from their modes, without destroying the photons in this time.

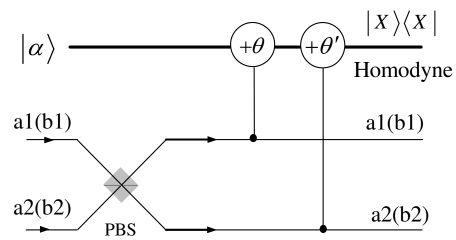

FIG. 6: Schematic diagram showing the principle of simple nondestructive quantum nondemolition detectors (QND$_3$) for purifying the photon pairs produced from two PDC sources. Each QND$_3$ is composed of two different cross-Kerr nonlinearities with the phase shift θ and θ', a PBS and a coherent laser probe beam $|\alpha\rangle$.

For a two-photon state without suffering from decoherence $(a_{1H}^+b_{1H}^+ + a_{1V}^+b_{1V}^+ + a_{2H}^+b_{2H}^+ + a_{2V}^+b_{2V}^+)|0\rangle$, the two parties Alice and Bob will get the same phase shift on their coherent states as QND$_3$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & (a_{1H}^+b_{1H}^+ + a_{1V}^+b_{1V}^+)|0\rangle|\alpha e^{i\theta}\rangle_a|\alpha e^{i\theta}\rangle_b \\ & +(a_{2V}^+b_{2V}^+ + a_{2H}^+b_{2H}^+)|0\rangle|\alpha e^{i\theta'}\rangle_a|\alpha e^{i\theta'}\rangle_b, \qquad (17)\end{aligned}$$

where $\theta\neq\theta'\oplus 2\pi$. If Alice and Bob get the same results with an X homodyne measurement (θ or θ'), they get a photon pair in the state $(a_H^+b_H^+ + a_V^+b_V^+)|0\rangle$. The homodyne measurement provides not only the information about the polarization state of the photon pair but also their spatial modes. If a bit-flipping error takes place, i.e., the state of the pair becoming $(|a_1\rangle|b_1\rangle+|a_2\rangle|b_2\rangle)(|V_a\rangle|H_b\rangle+|H_a\rangle|V_b\rangle)$, Alice and Bob will get two different results with their homodyne measurements on their coherent states $|\alpha\rangle$ as QND$_3$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & (a_{1V}^+b_{2H}^+ + a_{1H}^+b_{2V}^+)|0\rangle|\alpha e^{i\theta}\rangle_a|\alpha e^{i\theta'}\rangle_b \\ & +(a_{2V}^+b_{1H}^+ + a_{2H}^+b_{1V}^+)|0\rangle|\alpha e^{i\theta'}\rangle_a|\alpha e^{i\theta}\rangle_b. \qquad (18)\end{aligned}$$

One will get the result θ and the other θ'. By performing a bit-flipping operation $\sigma_x=|H\rangle\langle V|+|V\rangle\langle H|$ on one photon such as the photon controlled by Alice, Alice

and Bob can get rid of all bit-flip errors and obtain their uncorrupted pair $(a_H^+ b_H^+ + a_V^+ b_V^+)|0\rangle$.

For the four-photon state $(a_{1H}^+ b_{1H}^+ + a_{1V}^+ b_{1V}^+ + a_{2H}^+ b_{2H}^+ + a_{2V}^+ b_{2V}^+)^2|0\rangle$, the QND$_3$ detectors evolve the combined system to

$$\begin{aligned}\rightarrow\ & (a_{1V}^+ b_{1V}^+ + a_{1H}^+ b_{1H}^+)^2|0\rangle|\alpha e^{i2\theta}\rangle_a|\alpha e^{i2\theta}\rangle_b\\ & +(a_{2H}^+ b_{2H}^+ + a_{2V}^+ b_{2V}^+)^2|0\rangle|\alpha e^{i2\theta'}\rangle_a|\alpha e^{i2\theta'}\rangle_b\\ & +2(a_{1H}^+ b_{1H}^+ + a_{1V}^+ b_{1V}^+)(a_{2H}^+ b_{2H}^+\\ & +a_{2V}^+ b_{2V}^+)|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b. \qquad (19)\end{aligned}$$

Alice and Bob only pick up the four-mode instances, i.e., they get the same phase shift $\theta+\theta'$. If a bit-flipping error takes place on one of the two photon pairs in the four-photon state, i.e., the state of the two photon pairs becoming $(a_{1H}^+ b_{1H}^+ + a_{1V}^+ b_{1V}^+ + a_{2H}^+ b_{2H}^+ + a_{2V}^+ b_{2V}^+)(a_{1V}^+ b_{1H}^+ + a_{1H}^+ b_{1V}^+ + a_{2V}^+ b_{2H}^+ + a_{2H}^+ b_{2V}^+)|0\rangle$, the four photons pass through only three ports [27] after PBS. In this way, Alice and Bob can not get the same phase shift $\theta+\theta'$ when they measure their coherent states. When bit-flipping errors take place on both photon pairs in the four-photon state, Alice and Bob will get the same phase shift $\theta+\theta'$ as QND$_3$ detectors evolve the combined system from the state $(a_{1V}^+ b_{1H}^+ + a_{1H}^+ b_{1V}^+ + a_{2V}^+ b_{2H}^+ + a_{2H}^+ b_{2V}^+)^2|0\rangle|\alpha\rangle_a|\alpha\rangle_b$ to

$$\begin{aligned}\rightarrow\ & (a_{2H}^+ b_{1V}^+ + a_{2V}^+ b_{1H}^+)^2|0\rangle|\alpha e^{i2\theta'}\rangle_a|\alpha e^{i2\theta}\rangle_b\\ & +(a_{1V}^+ b_{2H}^+ + a_{1H}^+ b_{2V}^+)^2|0\rangle|\alpha e^{i2\theta}\rangle_a|\alpha e^{i2\theta'}\rangle_b\\ & +2(a_{1V}^+ b_{2H}^+ + a_{1H}^+ b_{2V}^+)(a_{2H}^+ b_{1V}^+\\ & +a_{2V}^+ b_{1H}^+)|0\rangle|\alpha e^{i(\theta+\theta')}\rangle_a|\alpha e^{i(\theta+\theta')}\rangle_b. \qquad (20)\end{aligned}$$

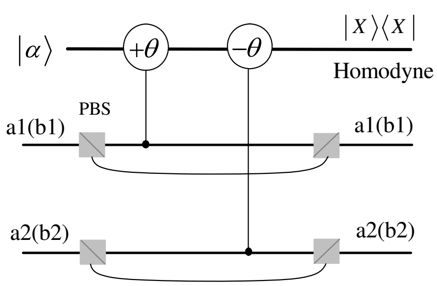

FIG. 7: The principle of the nondestructive quantum nondemolition detectors (QND$_4$) designed by Nemoto and Munro for parity check [28]. Two different cross-Kerr nonlinearities have the phase shift θ and $-\theta$, respectively.

From the discussion above, one can see that QND$_3$ detectors act as the same role as QND$_1$ if the two parties can control accurately the overlap time of the photons coming from the upper mode $(a_1(b_1))$ and the lower mode $(a_2(b_2))$. In experiment, each party need only use two different cross-Kerr nonlinearities, not four, which may make it more convenient than the QND$_1$ detectors.

In this entanglement purification based on ideal sources in Sec. III we do not exploit the QND detector designed by Nemoto and Munro [28] (namely QND$_4$ shown in Fig.7) as the QND$_2$ is more efficient than the latter. If Alice and Bob exploit QND$_4$ detectors to purify the two photon pairs produced by two ideal sources, they should perform a sophisticated X quadrature measurement in which the states $|\alpha e^{\pm\theta}\rangle$ can not be distinguished [28, 32] as QND$_4$ evolve the combined system from the state $(|H\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2} + |V\rangle_{a_1}|V\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2} + |H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2} + |V\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2})|\alpha\rangle_a|\alpha\rangle_b$ to the state

$$\begin{aligned}\rightarrow\ & (|H\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2}\\ & +|V\rangle_{a_1}|V\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2})|\alpha\rangle_a|\alpha\rangle_b\\ & +|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2}|\alpha e^{i\theta}\rangle_a|\alpha e^{i\theta}\rangle_b\\ & +|V\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2}|\alpha e^{-i\theta}\rangle_a|\alpha e^{-i\theta}\rangle_b. \qquad (21)\end{aligned}$$

This measurement can not be accomplished in a deterministic way, just in a nearly deterministic way. That is, Alice and Bob can not obtain the state $|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2} + |V\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2}$ perfectly, which is different from that with QND$_2$ in Sec. III.

In summary, we propose a different purification scheme based on two PDC sources with cross-Kerr nonlinearities. The task of entanglement purification can be completed with two steps in this scheme. First, we provides a primary entanglement purification protocol for PDC sources with QND detectors by transferring the spatial entanglement of photon pairs to their polarization. In this protocol, the QND detectors act as not only the role of CNOT gates but also that of photon number detectors, which provides a good way for the next process to purify the entanglement of the photon pairs more as they make the photon pairs equivalent to those coming from two ideal sources. Compared with the Simon-Pan protocol for PDC sources [27], this protocol does not require sophisticated single-photon detectors and can distinguish the number of the photons coming from the four modes. This advantage makes the two parties have the ability to complete the entanglement purification perfectly. In the second process for entanglement purification, new QND detectors are designed to act as the role of CNOT gates. This protocol does not require CNOT gates based on linear optical elements, but possesses the same yield of photon pairs purified as the protocols [16, 17] with CNOT gates, double that of the PBS protocol [18]. As a perfect CNOT gate is far beyond what is experimentally feasible with linear optical elements, this protocol may be an optimal one.

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